Life prediction of structures

Numerical study and treatment of structural durability, a brief report.

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Summary

This work is a brief report about the problem of life –or, durability study- of structural materials, undergoing mechanical, thermal and chemical actions. This document is approached from numerical techniques and, it highlights the potential of these types of tools in the study of structures undergoing very complex and coupled phenomena.

Keywords

Numerical methods, finite elements, fatigue, damage and degradation, life and durability of materials.

Brief report on the phenomena which reduce the life of structural materials life. Durability

Improvements made in the industrial performance of production methods for structures require new prediction techniques, able to make these methods more reliable, faster and cheaper. It is important to say that future responsibilities on structures oblige the manufacturer to demand more reliability in the obtained product. The concept "reliability" must be extended in time, that is to say, it is important to guarantee the "durability" of structural materials. A priori this demand of "reliability" and "durability" is very complex and difficult to guarantee. But nowadays there are simulation tools of the behaviour for "life prediction" and "integrity" throughout the life of the structural part, based on both numerical techniques and classical mechanics. An example of this tool is represented by the "Finite Elements Method" (FEM) -Zienkiewicz,Taylor (1998), Oñate (1992)- and its numerous applications to the study and control of mechanical processes.

Structural durability must be specified, depending on the type of structural element and its use. Thus the same integrity in time and under any action cannot be demanded for every type of structure. The definition of the structural durability period is established by the security required through market demands, and it goes against structure production costs. Therefore it is important to deepen the investigation and development of these tools, which are able to guarantee -qualitatively both and quantitatively- the integrity of a structural part during an established period and, in order to provide the service for which they have been designed. Generally, structural durability depends on different situations:

- Correct solutions to geometrical structural design problems in order to reach the ideal form. This problem is tacked through different optimisation numerical techniques, from the definition of an objective function, whose minimisation, regarding design variables gives the answer to the sought ideal form.

The degradation control in the material produced by mechanical, thermal, moisture and chemical actions whether predicted or net during the structural lifetime. These problems are caused by the following actions:

- Static and/or dynamic factors (impacts, vibrations, wind loads, quakes, differential motions, etc.);
- High or low temperatures (cryogenisation, fire action, etc.);
- Humidity and chemical aggressions (metals oxidation, aluminosis and the concrete carbonation, etc.);
- Radiation action and effects of matter decomposition (U.V radiation in composites, etc.)

All these situations cause structural degradation, such as microcracks, fractures, fatigue and distortions in the inner structure of the material. They must be studied from the point of view of mechanical phenomena of fractures, degradation, plasticity, viscoelasticity, fatigue and chemical-and-mechanical damage.

All these complex situations concurrence, developed in a material and caused by the aforementioned causes, endanger the material durability and in an indirect way, the structure. Numerical techniques, such as the finite elements method, are the few available tools in design, control and life prediction which guarantee security and good structural operation during the expected lifetime of that part.

Techniques for the durability study

The industry is always trying to know everything about materials durability, and especially about those materials meeting structural functions. Before we used to have a qualitative idea about life or durability of structural materials and conclusions were based on occurred happenings and errors. However it is sure that this way of studying does not guarantee nor satisfy, neither the industry nor the consumer, thus the certitude about durability is subsequent to the structure use. All this has leaded us to do quantitative studies, shedding more light on structural durability. Thus we could say, two study techniques have arisen:

- One of them based on experimental studies in laboratories;
- The other one based on previous simulations through numerical techniques.

In principle it can be said that both techniques are complementary. However numerical studies are becoming more and more important, thus they are cheaper, faster to do than the others and lately very reliable. Find hereafter a small presentation of both proceedings

Study methods and evaluations based on laboratory tests

The first scientific way of facing up the study of structural durability was through laboratory studies (Whöler 1871, Forrest 1962, and Osgood 1982). The results obtained are essential to know the behaviour and durability of the material, and they are as well the bases of the study and parameterisation of analytical and numerical methods, an essential character for the experimental study. Nevertheless the type of study to be done must be always and objectively analysed, as well as the result, which may be obtained by the experimental technique.

From a fast analysis can be emerged that the results obtained by the experimental way can not be easily extrapolated to the reality to be studied. Furthermore the high cost of an experimental study and the limited observation conditions, offered by a laboratory in order to reproduce the behaviour of real structures in time, does not allow this methodology to obtain every incontestable and secure result, which is demanded by the industry and the consumer.

From a small presentation of the problem, it is deduced that it is essential to add another complementary mechanical and numerical study to the experimental study. All of this allows extending the laboratory conclusions to the real problem, from solid basis, supported on the mechanics laws and used through numerical techniques. In short it can be said that time simulation, life prediction or durability of a real structure need the support of the following basis:

- Laboratory tests about materials and structural parts;
- Continuum mechanics;
- Numerical techniques, especially the finite elements method

Since this work is focused on the presentation of the mechanical and numerical study of the problem of structural durability, we abandon this section regarding experimental techniques, in order to mention the numerical techniques of study, basically based on the finite elements method.

Methods based on numerical techniques

As aforementioned, mechanical and numerical studies let us do a "conceptual extrapolation" of all phenomena observed in the laboratory. There are different types of numerical methods in order to approach the mechanics to the resolution of a real problem. Some methods such as, finite differences are based on the approximation of differential equations of mechanics; and some other methods, such as the finite elements method, focused to approach those unknown fields established by the laws of mechanics. It is obvious that these methodologies do not resolve the problem by themselves, thus they just establish the "vehicle" on which all mechanics formulations will be introduced. In the particular case of the study of structural durability, the numerical method is focused to manage the coupling among the different mechanical formulations:

- Behaviour law of microcracking and fracture
- Behaviour law of plasticity in metals and extension of formulation to non-metallic problems
- Behaviour law of degradation and structural damage
- Viscoplastic and viscoelastic behaviour law
- Behaviour law to sensitise the laws of the mechanics to the metals and non-metals fatigue problem
- Laws, establishing the chemical and mechanical coupling in metallic and non-metallic materials.

The Finite Elements Method (FEM) as an approximation technique of non-linear behaviour of structures

The FEM is a very appropriate method to solve many problems in mechanics. From the user point of view, its success lies on many virtues and few defects. The following advantages could be emphasized as the most important ones:

- Facility of the FEM to approach the variables of mechanics in complex domains
- Facility to incorporate real boundary conditions of the problem to be resolved
- Form of computational implementation (approved and efficient)

Everything has been possible thanks to the strong and fast development of computers during the last twenty years. If this had not occurred then, it would not be possible to present today the Finite Element Method as an evaluation tool of structural behaviour, used in the industry.

It is hard to determine the origins of the finite element method, thus there are very similar mathematics techniques already established on the second part of the 19th century, such as the Ritz method (Zienkiewicz, Taylor 1998). However the FEM, used as a tool for calculus in engineering, is more recent and it could be established that Courant implanted this technique in 1943 and used it in order to solve the twist problem of Siant-Venant. Thus fixing the origin of time for future engineering applications. 1953, Argyris, Turner, Clough, Martín and Topp sign the first work under the name of "Finite Elements Method", using the basis of the Ritz method. At the same time another works appeared, such as those developed by Zienkiewicz, Melosh, Tocher, etc., today known as the pioneers in the use of the FEM in mechanics problems solution and especially in the solution of structural and geotechnical problems. Find hereafter the main differences between the Ritz method and the FEM (Zienkiewicz 1994):

- Ritz: it uses an approximate formulation of the incognita field in the whole domain. This incognita must be in conformity with the whole domain.
- FEM: it uses an approximate formulation of the incognita field for one part of the domain, known as "finite element". This function must be in conformity with the finite element and must fulfil certain continuity conditions among the adjacent elements.

Figure 1 – Schematic representation of a global domain and its subdivision in local domains (see page 88).

Finite Elements Method (FEM) – Subdivision of the domain

The "condition of displacements compatibility" in a mechanical formulation, applied to the resolution of structures, lead to a method, where the incognita is equal to the load. As an alternative the "condition of equilibrium among forces", can be required, obtaining a formulation, where the incognita is equal to the displacements. These two ways of presenting formulation in order to resolve structures can be treated by the FEM, and even a new mixed formulation form may be proposed. In spite of these possible formulations and, due to the simplicity in the treatment of edge conditions, we often used the FEM in the context of displacements formulation. Once we the incognita field is defined, the FEM establishes a proceeding of functional-scale approximation in two different levels:

- Elementary-scale approximation: definition of functional approximation in a finite domain, within the global domain (see figure 1);
- Global or structural-scale approximation: imposition of compatibility and equilibrium conditions among the elementary domains (see figure 1).

Finite Element Method (FEM) – Definition of elementary and global equilibrium

The equation of mechanical equilibrium of a solid, undergoing external and thermal-mechanical actions, can be directly obtained through the first law of thermodynamics (Malvern 1969, Lubliner 1990). The objective is to obtain the displacements ui (x,y,z), that a structure is suffering, undergoing external actions (loads, temperature variation, etc.). In order to solve the problem, the choice would be to approach these functions through standarized polynomial as Njk (x,y,z), known as form functions (Zienkiewicz, Taylor 1998, Oñate 1992).

$$u_j(x,y,z)\big|_{\Omega^e} = N_{jk}(x,y,z)\ U_k\big|_{\Omega^e}$$

The following form function N jk (x,y,z), defined in the domain of a finite element, allows to approach the displacement fields u k (x,y,z) through the valuation of U_k in a finite number of points, known as nodes, which remain at the Ω^e domain.

From a mechanical-numerical point of view, the non-linearity in the equilibrium equation can be originated by different phenomena:

- **Constitutive non-linearity**, resulting from the loss of linearity between the stress and deformations fields It demands a special definition of a law material behaviour, just as occurs in plasticity, damage, fracture, fatigue, etc.
- **Non-linearity by big deformations**, due to the non-linear influence of the configuration change of the solid in the field of deformations. Furthermore these configuration changes are produced by big movements, translations and spins. They also produce changes in the local reference system in solid states, affecting therefore the compatibility deformation tensor.
- **Non-linearity due to big displacements**, in contrast to big deformations, it only affects to the compatibility deformation tensor B ijk because, due to big movements changes are just sensed in the local reference system of the solid points.

The equation of equilibrium at finite element Ωe participates in the global domain Ωthrough the "assembly operation" A, which represents an "ordinate sum" among the components of the force, bearing in mind the position and direction of the elementary forces.

In case of linearity in the solid behaviour, the following relation of global equilibrium is achieved, resulting from the assembly of the equations of local equilibrium, where

$$0 = \underset{\Omega^e}{\mathrm{A}}\left[f_k^{\mathrm{mas}} + f_k^{\mathrm{int}} - f_k^{\mathrm{ext}}\right]_{\Omega^e} = \Delta f_k\big|_{\Omega}$$

$f_k^{\mathrm{int}}\big|_{\Omega^e} \cdot f_k^{\mathrm{mas}}\big|_{\Omega^e}$ y $f_k^{\mathrm{ext}}\big|_{\Omega^e}$ represents the ordinate sets, as a column matrix of the internal, external and mass energy forces, developing in each point of the discrete system approaching the global. Any kind of non-linearity within the solid, become apparent through a unbalance among the internal and external forces, ΔfklΩ, which on a certain time instant "t" can be eliminated through the linearity of this unbalanced force, near the estate of present equilibrium.

The equation of dynamic equilibrium in a hard solid is usually seen in the following matrix form (S. Oller 2001):

$$0 = {}^{i+1}[\Delta f]^t_\Omega \cong {}^{i}[\Delta f]^t_\Omega + \underbrace{{}^{i}\left[M\frac{\int\ddot{U}}{\int U} + K^T + D^T\frac{\int\dot{U}}{\int U} - \frac{\int f^{ext}}{\int U}\right]^t_\Omega}_{{}^{i}J^t_\Omega}\ ?^{i+1}[\Delta U]_\Omega$$

This equation represents the equilibrium of forces, participating in the structure and linearity within the field of displacements. J $|J^T|_\Omega$ is the so-called Jacobs matrix; $[K^T]_\Omega$ represents the tangent stiffness matrix, $[M]_\Omega$ is the mass matrix, $[D^T]_\Omega$ is the damping matrix. All of them defined in the whole domain _ and, Ctijst is the tangent tensor corresponding to the constitutive law used in every point of the solid. The unbalanced force in the solid Ω y C^T_{ijst} can be eliminated, following the Newton-Raphson method until this residual is depreciable. This situation is known as convergence of the linearized process towards the exact solution. To deepen in the study, it is recommended to consult the book Zinkiewicz, Taylor (1998).

Non-linearity of materials – Inclusion of laws of materials behaviour in the FEM for the study of durability.

The law of solid behaviour in a point (Numerical Integration Point) is introduced in the equation of equilibrium (3), through a formulation, known as constitutive law. This formulation defines the stress sij the constitutive tensor Cijkl, the viscosity îijkl and the influence of kinematic non-linearity. All this allows to do a numerical and mechanical study, with a rational analysis of prediction of the structure behaviour in time, and therefore it allows us to study the structures durability.

The loads and time influence produces in some structural solids irrecoverable behaviours. Basically, three types of non-linear behaviours, dependent on time can be established:

- **Viscous elasticity or creep:** a deformation growth takes place at a constant applied stress (see figure 2).
- **Stress relief:** a lost of stress is produced, while the level of deformations remains the same. This behaviour - although non-reversible- represents the inverse implicit form of the creep (see figure 2).
- **Visco-plasticity:** its non-linear behaviour is due to a growth of the field of inelastic deformations. However this occurs, whenever the stress field goes beyond certain established limits (see figure 4).

There are also materials, which non-linear behaviour is independent to time (Malvern 1969, Lubliner 1990, Oller 2001). This situation can be a consequence of different phenomena, such as:

- **Plasticity or behaviour with immediate flow.** This behaviour can mathematically be established as a particular case of viscoplastic behaviour, although the physics used in the problem is qualitatively different (see figure 3).
- **Damage** or stiffness degradation: it produces a resistance loss in materials, due to the material elasticity degradation.

These behaviours can be presented, either isolated or all at the same but in a different degree. The ideal treatment of these phenomena alert about the materials behaviour to couple phenomena, such as:

- **Fatigue**, conceptual extension of plasticity and damage, influenced by the loads cycles (Suero, Oller 1998, ooler, Suero 1999, Salomón, Oller, Car, Oñate 1999, DARCAST 2000, González Torre 2001.
- **Physicochemical influence**: just as in the degradation of the material properties in moisture presence. An example of this, is the aluminosis effect and concrete carbonation in presence of moisture, which can be borne in mind in the constitutive definition as a conceptual extension of plasticity and damage, and whose formulations can include the definition of a chemical potential (Bosh 1998; Car, Oller, Oñate 1998).

Concerning the constitutive modelling and its influence in the behaviour of structures, an study must be done, deepening in concrete sources, specialised in each area. Nevertheless find in figures 3, 4 and 5 a definition of basic formulations, which can be used in order to study the material durability (fatigue, fracture, chemical-and-mechanical coupling).

Figure 2 – Simplified forms to understand the constitutive viscosity models of Kelvin and Maxwell (see page 90).
Figure 3 – Simplified forms of understanding the elastoplastic behaviour and damage (see page 91).
Figure 4 – Simplified forms of understanding the viscoplastic behaviour (see page 91).

Examples for numerical simulation of structural durability

Fatigue proof of a mould for solidification of aluminium parts

A steel mould for aluminium injection is presented. It will be represented by a fourth of the symmetric part. This mould works under an imposed cycle of temperatures and high pressures, that is to say, a mould with a coupling thermo-mechanical behaviour. The mould life has been studied for the imposed loads, and therefore a problem of fatigue has been solved through a constitutive damage model, influenced by the number of cycles (Oller S. 2001). Find hereafter a summary with the geometry, approaching of the finite net and materials properties (Salomón O., Oller S., Car E., Oñate E. 1999).

Material properties, geometry and finite elements net

Elements: 280; Nodes: 909; Quadrilaterals 8 nodes, with 4 Gauss points.

Material characteristics:
Steel undergoing micro-cracks behaviour, through an isotropic damage model.

Material characteristics:
Aluminium, undergoing flexible-isotropic damage.

Load characteristics:
Temperature imposition in the aluminium, following the next history: Pressure imposition on the mould sides in contact with the cast iron

Temperature evolution, acting on the side touching the injected aluminium.

Two load intervals are considered: the first one of 24 cycles of 45 seconds each, with increments on 5 seconds (the temperature load starts at 200°C and reaches 690°C, when the aluminium is injected. Then when the aluminium part is extracted, it is reduced to 230°C and, finally the cycle finishes when the mould reaches a temperature of 200°C). During the second state of loads, the temperature stays at the maximal temperature reached at the end of the first interval and, the increase is considered in number of cycles, instead of time.

Fatigue proof in one part (made of aluminium) of the alternating current generator

We introduce in this chapter a numerical and experimental analysis about the fatigue problem in an aluminium part for alternator support (as the one shown in next figure). The Polytechnic of Turin has developed the experimental study, where they have put the part under a cyclical load in its extreme, as shown in next figure. CIMNE (DARCAST 2001) has developed the test in this numerical part. Around 50 parts have been used for the fatigue test, in order to obtain reliable experimental conclusions. Once the experimental test in finished, several radiographic analysis have been done in order to know the porosity state in the inside of the part and the fractures surfaces have been analysed through metalographic studies. All this has permitted to adjust the parameters of the material and do a numerical study of verification through finite elements. Photograph this page, made of aluminium. Also shown the way to apply the load in order to develop the fatigue test. Find hereafter the form in which the cyclic load is applied by finite elements in the analysed part.

See hereafter the coincidence between experimental and numerical results.

Numerical Analysis of Life Prediction in hydraulic mini-components

Throughout this article we will see a numerical simulation by finite elements (FEM), in order to do a useful life prediction of "hydraulic mini-bombs", undergoing high-pressure cycles. These numerical results have been developed within the European project MINIHAP (2001), concerning manufactured bombs, tested in laboratory by the company Roquet, S.A. The bombs have been manufactured with two different materials: 7003 Eural-type aluminium and GGG-40 globular iron-cast.

CIMNE has numerical studied the life prediction of these hydraulics bombs. They have worked using a 3D geometry on real plans and, a cyclical interior pressure has been imposed (see following figure). The material properties have been considered depending on data, offered by the manufacturer.

Through numerical simulation, CIMNE has obtained an excellent approach to the experimental test both, in localising the point of maximal fatigue (see figure) and reaching this state number of cycles. The experimental test shows that the fatigue fracture is reached at 75881 cycles, and the numerical test confirms this data at 80000 cycles.

Find hereafter the curves "Stress-Number of cycles" and "Damage-Number of cycles". It can be observed how resistance falls as damage increases, when the highest damage point by fatigue is reached at 80000 cycles of pressure in the interior of the bomb.

Conclusions

The continuous methods mechanics, assisted by numerical techniques, and especially the finite element method, constitute a very strong tool for the study and life prediction of parts, accomplishing a structural function. In this article we have done an introduction, showing the basis of this technique and presenting several examples with the wide possibility of this method. References are given in order to complete the reading and to acquire a clearest idea about the possibilities offered by this way of work.

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